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| 8 | a) | The probability that a student in a certain college owns a smart phone is 0.8. Calculate the Probability that the 7th student randomly interviewed in that college is the 5th one to own a smart phone. |
|  | b) | A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years? Use a 0.05 level of significance. |
|  | c) | The heights of a random sample of 50 college students showed a mean of 174.5 centimeter and standard deviation of 6.9 centimeters. Construct a 98 % confidence interval for the mean of all college students. |
| 9 | a) | Past experience indicates that the time required for high school seniors to complete a standardized test is a normal random variable with a standard deviation of 6 minutes. Test the hypothesis that  against the alternative that  if a random sample of the test times of 20 high school seniors has a standard deviation. Use a 0.05 level of significance. |
|  | b) | The grades in a statistics course for a particular semester were as follows:   |  |  |  |  |  | | --- | --- | --- | --- | --- | | Grade | A | B | C | D | | f | 24 | 18 | 32 | 26 |   Test the hypothesis, at the 0.05 level of significance, the distribution of grades in uniform. |
|  | c) | Consider a Poisson distribution with probability mass function  Suppose that a random sample  is taken from the distribution. Evaluate the maximum likelihood estimate of . |
| 10 | a) | A study of the amount of rainfall and the quantity of air pollution removed, produced the following data:   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | Daily rainfall (in 0.01 cm) x | 4.3 | 4.5 | 5.9 | 5.6 | 6.1 | 3.8 | | Particulate Removed (µ g/m3)  y | 126 | 121 | 116 | 118 | 114 | 132 |   Calculate the correlation coefficient. |
|  | b) | For the Data given in 10(a) ,Deduce the regression line of Yon X |
|  | c) | Using the regression line, estimate the amount of particulate removed when the daily rainfall is 4.8 units. |

**ODD SEMESTER EXAMINATION, DECEMBER-2019**

**PROBABILITY & STATISTICS (MTH-2002)**

**Programme: B.Tech Semester : 3rd**

**Full Marks: 60 Time: 3 Hours**

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| **Subject/Course Learning Outcome** | **\*Taxonomy**  **Level** | **Ques.**  **Nos.** | **Marks** |
| Apply probability axioms to compute probability and conditional probability | L3,L3,L4,L3 | 1(a,b), 2(a) | 2\*3 |
| Define random variables and compute probability distributions, joint & marginal distribution | L4,L4,L3,L5,L5 | 2(b,c), 3(a,b,c) | 2\*5 |
| Compute expectation of random variables and their functions and compute moments and moment generating functions of a random variable | L3,L4,L4  L3,L4 | 4(a,b,c),6(a,b) | 2\*5 |
| Discuss discrete probability distribution viz: Binomial, Poisson & Hypergeometric and continuous probability distribution distributions viz: Uniform, Normal Gamma & Exponential | L3,L4, L4, L4 | 1(c),5(a,b), 6(c), | 2\*4 |
| Estimate the population mean and variance of a normal distribution by point and interval estimation | L4 | 7(b),5(c) | 2\*2 |
| Infer about population parameter through hypothesis testing with the help of a random sample | L1,L4,L3,L4,L4, L3,L4,L4 | 7(a,c) 8(a,b,c), 10(a,b,c) | 2\*8 |
| Analyze linear regression and co-relation | L3,L5,L5 | 9(a,b,c) | 2\*3 |

\*Bloom’s taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

**Answer all questions. Each question carries equal mark.**

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| --- | --- | --- |
| 1 | a) | An experiment consist of picking of 3 balls from bag containing 10 red and 10 black balls. Draw the tree diagram and list the elements of the sample space based upon the color only. |
|  | b) | If 4 books are picked at random from a shelf containing 7 mathematics, 3 physics and 2 chemistry books, then calculate the probability that 2 mathematics books are selected. |
|  | c) | The probability that an automobile being filled with gasoline also needs an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and the filter need changing is 0.14.  If the oil has to be changed, evaluate is the probability that a new oil filter is needed . |
| 2 | a) | Police plan to enforce speed limits by using radar traps at location L1, L2, L3 and L4 which will be operated 40%, 30%, 20% & 30% of the time respectively. If a person who is speeding on his way to work has probabilities 0.2, 0.1, 0.5 & 0.2, respectively, of passing through these locations, Compute the probability that he will receive a speeding ticket . |
|  | b) | With reference to 2(a), if the person received a speeding ticket on her way to work, what is the probability that she passed through the radar trap located at . |
|  | c) | Let X is a discrete random variable with probability function . Construct the cumulative distribution function of the random variable X. |
| 3 | a) | Compute the value of the constant ‘k’ used in the following probability distribution  . |
|  | b) | The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function.    Compute the probability density function ; also calculate the probability of waiting less than 12 minutes between successive speeders. |
|  | c) | Let X and Y denote the lengths of lifes, in years, of two components in an electronics system. If the joint density function of these variables is    Calculate the marginal distribution of X. |
| 4 | a) | If the joint probability distribution of X and Y is given by  . Compute |
|  | b) | If a dealer’s profit, in units of $5000, on a new automobile can be looked upon as a random variable X having the density function    Find the average profit per automobile. |
|  | c) | The random variable X, representing the number of errors per 100 lines of software code, has the following probability distribution:   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | X | 2 | 3 | 4 | 5 | 6 | | f(x) | 0.01 | 0.25 | 0.4 | 0.3 | 0.04 |   Calculate the variance of X. |
| 5 | a) | A random variable X has a mean  and a variance . Using Chebyshev’s theorem, find |
|  | b) | A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%. The inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20? |
|  | c) | A foreign student club lists as its members 2 Canadians, 3 Japanese, 5 Italians, and 2 Germans. If a committee of 4 is selected at random, find the probability that all nationalities are represented. |
| 6 | a) | The average number of field mice per acre in a 5-acre wheat field is estimated to be 12. Using Poisson distribution, calculate the probability that fewer than 7 field mice are found on a given acre. |
|  | b) | Suppose X follows a continuous uniform distribution from 1 to 5. Determine the conditional probability. |
|  | c) | A lawyer commutes daily from his suburban home to his midtown office. The average time for a one-way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assume the distribution of trip times to be normally distributed. Calculate the probability that a trip will take at least 30 minutes. |
| 7 | a) | A coin is tossed 400 times. Use the normal approximation to binomial distribution to calculate the probability of obtaining the number of heads between 185 and 210 inclusive. |
|  | b) | The life, in years, of a certain type of electrical switch has an exponential distribution with an average life . If 100 of these switches are installed in different systems, what is the probability that at most 30 fail during the first year? |
|  | c) | A random variable X has the discrete uniform distribution  Show that the moment-generating function of X is |